USE GAMES TO MOTIVATE YOUR CALCULUS STUDENTS

Robert E. Kowalczyk and Adam O. Hausknecht
University of Massachusetts Dartmouth
Mathematics Department, 285 Old Westport Road, N. Dartmouth, MA 02747-2300
rkowalczyk@umassd.edu and ahausknecht@umassd.edu

As mathematics instructors, we are always searching for new ways to motivate our students’ learning of precalculus, calculus and differential equations. How many times have we heard our students say: “When will we ever use this?” or “This is boring.” Some students find mathematics dry and do not fully engage themselves in classroom discussions and activities. To help these students (and all students in general), we developed a set of computer games that they play using mathematical models. These games add a spark of fun and enjoyment for our students in their pursuit of learning mathematics.

Warm-up Exercises

In a first-year calculus course, the instructor quickly discovers students’ weaknesses in using and understanding functions. Additionally, students’ “working” libraries of mathematical functions are frequently limited. We designed a set of games that encourage students to develop a better understanding of functions and their properties. One of these games, A Piecewise Plumbing System, is described below:

The object of this game is to find a pair of piecewise functions that trace the path of the front most pipe system going from a bathtub on the first floor to a boiler in the basement of the house. You are free to use any type of function to model each section of pipe. Game points are given for how well the piecewise model fits the pipes and bonus points are given for each additional different type of function used, for example, linear, quadratic, rational, algebraic, and so on. Find one piecewise function that models the upper portion of the pipes and a second piecewise function that models the lower portion of the pipes.

The learning objectives for this game are:

1. Find the equation of a line between two points.
2. Review polynomials, rational functions, and algebraic functions.
3. Make new functions from simple functions by shifting, stretching, or shrinking.

Figure 1 A Piecewise Plumbing System
4. Practice building and using piecewise functions.

The piecewise functions for the upper and lower portions of the pipes in this example are

\[
f = \begin{cases} 
0.6 \sqrt{x - 1.22} + 5.2 & 1.22 \leq x \leq 1.45 \\
0.19(x - 6.9) + 6.6 & 1.45 < x \leq 6.9 \\
1.5(x - 6.8)^2 + 6.58 & 6.9 < x \leq 7.45 \\
5.8(x - 7.66) + 8.5 & 7.45 < x \leq 7.7 \\
\end{cases}
\]

\[
g = \begin{cases} 
-4.55(x - 1.44) + 4.32 & 1.22 \leq x \leq 1.46 \\
0.5 \sqrt{x - 1.2} + 3.25 & 1.46 < x \leq 2 \\
-0.18(x - 4.4) + 3.36 & 2 < x \leq 4.4 \\
0.5 \sqrt{x - 4.4} - 3.36 & 4.4 < x \leq 4.8 \\
0.76 (x - 5.9)^3 + 0.76 & 5.24 < x \leq 5.74 \\
0.76 & 5.74 < x \leq 6.95 \\
\end{cases}
\]

and their graphs are shown in Figure 1.

**Starting the Game**

Once students have developed a good working library of functions, it becomes an easier task for them to apply the techniques of calculus. We have developed a set of games integrating data sampling, curve fitting, and calculus. The game, *Modeling Wing Aerodynamics of a Butterfly*, has the following directions:

The Aerodynamics Research Institute is studying the flight dynamics of butterflies. As part of their study, they need an accurate estimate of the wing area of a butterfly. You have been hired as a consultant to calculate the area of the Polyfit Morpho butterfly. After studying the problem description, you have decided to fit the wing with piecewise polynomial functions and to use calculus to find the area between the polynomial functions. Your problem solving strategy is to place points along the edge of the wing and to use TEMATH’s Least Squares Polynomial Fit tool (or the Interpolation tool) to find the

---

**Figure 2  Finding the Area of a Butterfly’s Wing**
coefficients of the polynomials. Once the polynomials have been determined, you will use TEMATH’s Integration of the Difference of Two Functions tool to find the area of the wing.

As is shown in Figure 2, piecewise polynomial functions provide an excellent fit to the shape of the butterfly’s wing and, hence, yields an excellent estimate of the area of the wing. The total estimated area of the four wing sections of the butterfly is 37.3.

Getting into the Game

In a first-year calculus course, we introduce students to the concept of parametric equations but have little time to give them a good experience in parameterizing a variety of curves. A fun way for a student to gain experience in parameterizing curves is to play the Parametric Racetrack game.

You and your friend are Mathcar race drivers. Your first objective is to find the parametric equations that draw the path of your car through the center of the track for one complete revolution. Now that you have taken your practice lap, you and your friend need to find the parametric equations for the following races:

1. You and your friend race to a photo finish after one lap. Your car races on the outside of the track and your friend’s car races on the inside of the track.

2. You and your friend race as described above but this time you beat your friend by a bumper. Additional points are given if you come from behind to beat your friend.

3. Your friend will mark a random point on the racetrack. You must now find the parametric equations that draw the path of your car starting at the random point and ending at the same random point after completing one revolution.

4. In the final “Ultimate Challenge” race, you and your friend both mark a random point on the racetrack. Your race begins at your friend’s point and ends at the finish line. Your friend’s race begins at your random point and ends at the finish line. The race must end in a photo finish.

You can use TEMATH’s tracker tool to trace the path of each car in the race, watch the race in action, and observe the exciting finish of each race.

Figure 3 Parametric Racetrack
An example solution to race 2 is shown in Figure 3. The parametric equations for this race are

Outer (Red) Path
\[
\begin{align*}
x(t) &= 3.8\cos\left(t^2/6\right) \\
y(t) &= 2.3\sin\left(t^2/6\right)
\end{align*}
\]

Inner (Blue) Path
\[
\begin{align*}
x(t) &= 3.65\cos(t) \\
y(t) &= 2.15\sin(t)
\end{align*}
\]

Saving the Game — An Environmental Comeback

In the game *Saving the Elephants — A Polar Solution*, students use polar equations to model the shape of a lake and to calculate the area of the lake. The description of the game is given below.

Hwange Lake in Zimbabwe, Africa is a favorite watering hole for elephants. In recent years, a plant not native to Zimbabwe has taken root in the lake and its rapid predatory growth has threatened to take over the lake and kill all native plants and amphibian life, thus killing the lake and its water source for the endangered African elephant. Your job as consultant to the Zimbabwe Natural Trust Council is to estimate the surface area of the lake. Notice that the lake is shaped like a familiar polar curve. Find the polar curve that best models the shape of the lake and use integration (or TEMATH’s Integration tool) to find the area enclosed by the polar curve, and hence, obtain an estimate for the surface area of the lake. The Zimbabwe Natural Trust Council can now order the correct quantity of an environmentally safe chemical that will be used to kill the invading plant species. Assume that the units of measurement are meters.

An example model is shown in Figure 4. The polar equation model is the cardioid

\[
r = 131 + 131\cos(t)
\]

and the estimate of the area is \(80,869 \text{ m}^2\).

Speeding to the Finish of the Game

We have also developed a set of games that are designed to help students better understand the qualitative properties of various differential equation models and their parameters. For example, in the game *The Logistic Golf Tournament*, students are asked to find
the modified logistic differential equation that would sink the last shot on the 18th hole. This differential equation model
\[ \frac{dy}{dt} = ky \left( \frac{y}{M} - 1 \right) \left( 1 - \frac{y}{N} \right) \]
has the parameters \( k, M, \) and \( N \). In trying to model the path of the golf ball to the hole, the student needs to develop an understanding of the three parameters and how their values affect the shape of the graph of the solution to the differential equation. Building an intuition of differential equation models and their parameters is the key to performing a qualitative analysis of differential equations and their solutions. Students use TEMATH’s Differential Equation Solver to enter their differential equation model and to plot its solution curve.

Figure 5 The Logistic Golf Tournament

TEMATH (Tools for Exploring Mathematics)

TEMATH (Tools for Exploring Mathematics) is a mathematics exploration environment useful for investigating a broad range of mathematical problems. It is effective for solving problems in precalculus, calculus, differential equations, linear algebra, numerical analysis, and math modeling. TEMATH contains a powerful grapher, a matrix calculator, an expression calculator, a differential equation solver, a facility for handling and manipulating data, numerical mathematical tools, and visual and dynamic exploration tools. TEMATH requires an Apple Macintosh computer (post MacPlus; PowerPC even better) running MAC OS 7.5-9.04, a 12" or larger monitor screen, 3 MB of free RAM, and 2MB of disk space (TEMATH plus files). You can download a copy of TEMATH and its documentation, application files, and games from:

www2.umassd.edu/TEMATH

Bibliography


[3] The game background pictures were drawn by Josh Allan, design student, University of Massachusetts Dartmouth, 2000.