An investigation on the breakup of underwater buoyant oil jets: computational simulations and experiments

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Abstract

We present experimental and computational results on the breakup of underwater buoyant oil jets and plumes at a wide range of Reynolds, Weber and Richardson numbers and viscosity ratios. The results show three main jet breakup regimes: atomization, skirt-type and pinch-off. The threshold Weber number for the atomization regime is around 100, which varies slightly with the jet Eötvös number. Furthermore, it is demonstrated that the correlation proposed by Masutani and Adams as the boundary for the atomization regime applies to our broader data set too. The experimental and computational results both suggest that in a buoyancy-driven jet breakup occurs only when the jet is accelerated to a point where the local Richardson number, defined based on properties at breakup, becomes less than 0.4, in which case the local Weber number is above 10. The computational results reveal the mechanisms leading to formation of small droplets around the perimeter of energetic jets and umbrella-shaped jet separations at less energetic cases. The time-averaged lateral expansion of the simulated jets, representing four different conditions, are presented as a function of the height along the jet. The computational results were obtained by using a GPU-accelerated MPI parallel two-phase flow solver, which provides acceleration factors between 3 to 6, compared to running on CPUs only.

Keywords: Underwater buoyant oil jets; Jet breakup; Primary atomization; Immiscible plumes; Underwater oil spill; GPU acceleration
1 Introduction

Underwater buoyant jets and plumes are encountered in nature and many engineering problems, including underwater volcanic eruptions, hydrothermal plumes, and underwater oil leaks during off-shore petroleum extractions. The latter is of particular importance because of increased number of off-shore oil drilling projects and rising potential of underwater oil spills, similar to the 2010 Deepwater Horizon oil spill disaster in the Gulf of Mexico, which covered 68,000 square miles of marine habitat. Taking three months to deliver, the mitigation efforts to contain the oil leak were drastically impeded, and even misguided in the case of the “Top Kill” procedure, by a lack of both understanding and reliable information on the oil jet gushing out 5000 feet underwater. Better understanding of the behavior of such underwater buoyant oil jets and the ability to predict their breakup patterns and lateral expansion would be of great value in development of plans to mitigate future underwater oil leaks.

Consider a fluid injected into an initially quiescent fluid of higher density, where the injection is in direction of the buoyant force. Three cases can exist based on the relative magnitude of the injection momentum and the buoyancy effects: the injection momentum can be much stronger than the buoyancy effects, in which case the flow is referred to as a “momentum jet” (or simply jet) in the literature. Or, the buoyancy can be the most dominant effect at the source and a “plume” is said to form. And, in a third case, a “buoyant jet” can exist, in which the flow is dominated by the injection momentum close to the source but soon becomes buoyancy-driven. For brevity, we will use jets or plumes to refer to the flow here.

The ratio of the buoyancy to momentum effects is given by the Richardson number (Ri) defined for underwater buoyant oil jets as

\[
Ri = \frac{g' D}{u^2} = \frac{g D}{u^2} \left( \frac{\rho_{\text{water}} - \rho_{\text{oil}}}{\rho_{\text{oil}}} \right)
\]

where \( D \) is the jet outlet diameter and \( u \) is the injection velocity. Note that the densimetric
Froude number, which is equal to $1/\sqrt{\text{Ri}}$, is also used frequently in literature to express the relative magnitude of the momentum to buoyancy effects. The Reynolds number (Re) at the jet outlet represents the ratio of momentum to viscous effects and is defined as

$$\text{Re} = \frac{\rho_{\text{oil}} u D}{\mu_{\text{oil}}}$$  \hspace{1cm} (2)

where $\mu$ is the dynamic viscosity. The Weber number (We), defined below, represents the ratio of momentum to surface tension effects for immiscible jets

$$\text{We} = \frac{\rho_{\text{oil}} u^2 D}{\sigma}$$  \hspace{1cm} (3)

where $\sigma$ is the surface tension coefficient. We also use the Ohnesorge (Oh) and Eötvös (Eo) numbers, defined as

$$\text{Oh} = \frac{\mu_{\text{oil}}}{\sqrt{\rho_{\text{oil}} \sigma D}} = \frac{\sqrt{\text{We}}}{\text{Re}}$$  \hspace{1cm} (4)

$$\text{Eo} = \frac{g (\rho_{\text{water}} - \rho_{\text{oil}}) D^2}{\sigma} = \text{Ri} \times \text{We}$$  \hspace{1cm} (5)

Oh represents the ratio of viscous forces to surface tension and Eo represents the ratio of buoyancy forces to surface tension.

The studies on plumes and turbulent jets that were performed prior to early 80s were reviewed by List [9]. More recently, Lipari and Standby [8] reviewed the experimental data on incompressible turbulent round jets. They considered swirl-free and neutrally-buoyant jets and reported reviews on far-field quantities such as mean velocity, Reynolds stresses and momentum flux conservation obtained by classic Reynolds-averaged measurements.

Masutani and Adams [11] conducted experiments on the evolution of underwater jets of crude oil discharged from small orifices (1, 2 and 5 mm in diameter). They investigated jets with $2 \lesssim \text{Re} \lesssim 1000$ and $1 \lesssim \text{We} \lesssim 100$ and identified two parallel drop generation mechanisms: surface instabilities generate small droplets, while large drops form due to the breakup of the jet core. However, no clear correlation between the droplet sizes and orifice diameter was identified. Identifying three different modes of breakup, from blob formation
(Rayleigh instability) to atomization, Masutani and Adams [11] proposed correlations between the jet Oh and Re that approximate the boundaries of breakup modes. Recently, using Particle Image Velocimetry (PIV), Guno et al. [7] presented time-resolved 3D velocity field and flow structures of a buoyant liquid jet of low Froude number (0.3) and relatively low Re (200), where the spiral vorticity field was clearly mapped out.

Funada et al. [6] performed stability analysis of a liquid jet issued into another liquid (or an incompressible gas) by using a “viscous potential” solution to the flow equations, where the viscous stresses enter at the fluid interfaces. Their analysis considered, among others, the effects of liquid jet viscosity and the density and viscosity of the ambient fluid. Extending previous analytical solutions, Michaux and Vauquelin [14] carried out a theoretical analysis of turbulent plumes and presented analytical expressions for plume radius, vertical velocity and density in the near field as well as the far field. These expressions apply to both Boussinesq and non-Boussinesq plumes in homogenous background environment. Mehaddi et al. [13] performed a theoretical investigation of turbulent plumes rising vertically in an ambient fluid that is linearly stratified. They presented analytical expressions for the height of a Boussinesq plume and the buoyancy and volume fluxes at top of the plume.

Yappa et al. [15] developed a model to simulate the concentration of gasses and hydrates released in deep waters. Integrating the hydrodynamics and thermodynamics models with the transport processes, they simulated the evolution of underwater plumes of gases and hydrates over large distances (e.g. several hundreds of meters). The model was an improvement on earlier models, and the results show good agreement with the experimental data on gasses released at a water depth of 1000 meters.

In this paper, we present computational simulations and experimental results on the breakup of underwater buoyant oil jets at a wide range of Re, We and Ri numbers and viscosity ratios. Three different jet breakup regimes are identified, and a threshold Weber number for atomization is proposed. Furthermore, we present how the Oh-Re correlation
proposed in [11] applies to a broader experimental data set. The interactions between the oil and surrounding water are demonstrated through the computational simulations and the vorticity field. The time-averaged lateral expansions of the jets under different conditions are also presented.

The paper is organized as follows: Section 2 reviews the governing equations and the boundary conditions of interfacial flows. In Section 3, the flow solver and an overview of its numerical methods are presented. Section 4 describes the experimental methods. The experimental and numerical results are presented in Section 5. Finally, some concluding remarks are given in Section 6.

2 Governing equations

Consider interfacial flows of Newtonian, immiscible and incompressible fluids. The governing equations for such flows are the conservation of mass and momentum, Equations (6) and (7), respectively,

\[ \nabla \cdot \mathbf{U} = 0 \]  
\[ \frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla p + \nabla \cdot \tau + \mathbf{F}_B + \mathbf{F}_{ST} \]

where \( \mathbf{U} \) denotes velocity vector, \( p \) pressure, \( \tau = \mu (\nabla \mathbf{U} + \nabla \mathbf{U}^T) \) the shear stress tensor, \( \mathbf{F}_B \) any body forces such as gravity, and \( \mathbf{F}_{ST} \) the surface tension force.

Three primary boundary conditions exist along fluid interfaces. When there is no mass interchange between the fluids, the normal components of fluid velocities are equal. Thus,

\[ \left[ \mathbf{U} \cdot \mathbf{n} \right] \equiv \mathbf{U}^{(1)} \cdot \mathbf{n} - \mathbf{U}^{(2)} \cdot \mathbf{n} = 0 \]  

where \( [ . ] \) gives the jump of a variable across the fluid interfaces. In addition, in viscous fluids, the velocity components tangent to the interface are equal too. Therefore,

\[ \left[ \mathbf{U} \right] = 0 \]
Denoting the surface tension coefficient by $\sigma$, assumed constant, the following jump condition exists in the stress tensor

$$\mathcal{L}((-pI + \tau) \cdot \hat{n}) = \sigma \kappa \hat{n} \quad (10)$$

In addition to Equations (6) and (7), the fluid interfaces are advected by solving the following transport equation in a volume-of-fluid (VOF) context,

$$\frac{\partial f}{\partial t} + U \cdot \nabla f = 0 \quad (11)$$

where scalar function $f$ is defined as

$$f(x) = \begin{cases} 1, & x \in \text{Fluid 1} \\ 0, & x \notin \text{Fluid 1} \end{cases} \quad (12)$$

The interface normal vector $\hat{n}$ and curvature $\kappa$ are calculated from $f$ via

$$\hat{n} = \frac{\nabla f}{|\nabla f|} \quad (13)$$

$$\kappa = -\nabla \cdot \left( \frac{\nabla f}{|\nabla f|} \right) \quad (14)$$

A single set of Equations (6) and (7) is solved in the entire domain, and the fluid properties at any point are determined by $f$ from

$$\rho(f) = f \rho^{(1)} + (1 - f) \rho^{(2)} \quad (15)$$

$$\mu(f) = f \mu^{(1)} + (1 - f) \mu^{(2)} \quad (16)$$

where $f$ represents fluid 1 with density $\rho^{(1)}$ and viscosity $\mu^{(1)}$.

### 3 Numerical method and the GPU-accelerated flow solver

We employ the MPI parallel two-phase flow solver originally developed at the University of Toronto’s Centre for Advanced Coating Technologies [1], in which Equations (6) and (7) are
solved by using the two-step projection method [3] in a finite volume context. Equation (7) is solved by splitting it into predictor and corrector steps

\[
\frac{\rho^{n+1} U^* - \rho^n U^n}{\Delta t} = -\nabla \cdot (\rho U U)^n + \nabla \cdot (\mu (\nabla U + \nabla^T U))^n + F^m_B + F^m_{ST}
\]

(17)

\[
\frac{\rho^{n+1} U^{n+1} - \rho^{n+1} U^*}{\Delta t} = -\nabla p^{n+1}
\]

(18)

where superscripts \( n \) and \( n + 1 \) denote the current and next time levels, and * an interim level. The surface tension \( F_{ST} \) in Equation (17) is implemented using the well-known CSF model [2]. Applying Equation (6) to Equation (18) yields an implicit equation for pressure,

\[
\nabla \cdot \left( \frac{1}{\rho^{n+1}} \nabla p^{n+1} \right) = \frac{1}{\Delta t} \nabla \cdot U^*
\]

(19)

where \( p^{n+1} \) is obtained by solving the pressure Poisson Equation (19) iteratively. Knowing \( p^{n+1}, U^{n+1} \) is then evaluated from Equation (18).

To accelerate the iterative solution to Equation (19), Codyer et al. [4] ported the pressure Poisson solver to Graphics Processing Units (GPUs) in a heterogeneous multi-CPU/GPU computing framework. The GPUs are solely used for the pressure Poisson solver, while the remaining calculations are performed on CPUs. The parallel flow solver decomposes the numerical domain into multiple sub-domains, where each MPI sub-domain is assigned to a single CPU processor core and one GPU card. The sub-domains communicate shared data across boundaries; there is no direct communication between the GPUs. Compared to running on CPUs, by using GPUs, MPI parallel simulations are completed between 3 to 6 times faster.

Finally, fluid volumes are advected by solving Equation (11) geometrically using the mass-conserving VOF method of Youngs [16]: fluid interfaces are reconstructed at each timestep using the known volume fractions at time \( n \). Then, volume fluxes across control volume faces are evaluated geometrically, leading to \( f^{n+1} \). Note that the simulations presented here use a 3D Cartesian grid that is uniform in all directions.
4 Experimental method

Positively buoyant jets and plumes were directed vertically upward into an immiscible surrounding fluid using the apparatus shown in Figure 1. The equipment, which is based on a modified apparatus discussed in [5], consisted of a 30 cm (L) × 30 cm (W) × 70 cm (H) test chamber, a circulating pump loop and an illumination and data acquisition system. The circulating loop drew the fountain fluid from a reservoir at the top of the test facility, and pumped it through a throttle valve and flow meter into a cylindrical pipe with diameter ranging from 0.90 cm to 2.38 cm (nominal pipe diameters: 1/4”, 3/4” and 1”). The facility was illuminated with a light sheet generated using the beam of a New Wave Research dual-head, Solo 30 PIV Nd:YAG passed through a pair of cylindrical lenses that form a light sheet and a spherical lens that controlled the light sheet thickness. Digital images were captured using a TSI Model# 630057 PowerView plus 2 Megapixel digital CCD camera. The fluid flow structure was analyzed using TSI Insight 6 particle image velocimetry (PIV) Software. The jets/plumes consisted of silicone oil of varying viscosity, while the ambient fluid consisted of a glycerin/water mixture, mixed in a ratio to match the refractive index of the fluid pairs following the methodology of [10]. The experimental data set consists of 82 different runs, selected from three pipe diameters, three fluid viscosity combinations and varying flow rates as summarized in Table 1. Three to five images were examined for each of the 82 experimental runs.

5 Results

5.1 Experimental results

Figures 2, 3 and 4 show experimental images of oil jets discharged into water from 0.90, 1.83 and 2.38 cm pipes, respectively. On each figure, the top row corresponds to 0.65 cSt oil, the middle to 5.0 cSt and the bottom to 20 cSt (see Table 1). The flow rate is the same on each
column and increases from left to right. Each of the experimental runs was classified by flow regime based on the characteristics of the flow structure and breakup. We defined three flow regimes as shown in Figure 5. Regime 1, defined as the primary atomization regime, is the most energetic breakup regime. Here, the plume or jet breaks into droplets smaller than the plume diameter a short distance from the onset of breakup. Regime 2, the non-atomization regime, is subdivided into the pinch-off and skirt regimes. The pinch-off regime (regime 2A) results when an instability in the jet results in a reducing diameter that separates droplets that are greater than the diameter of the jet. The skirt breakup regime (regime 2B) occurs when the pinch off point rolls up to form a skirt.

Although the data are divided roughly into an atomization and non-atomization area, Figure 6 reveals no correlation of the breakup regime as a function of either Ri or Re. Figure 7, on the other hand, demonstrates interfacial tension effects in both We and Eo numbers with the Weber number effects being dominant. Atomization onset occurs at Weber numbers of around 200 at low Eötvös numbers and decreases to Weber numbers of around 80 at higher Eötvös numbers. This suggests that the interfacial tension is acting to suppress atomization while a combination of inertial forces (strong correlation) and buoyant forces (weak correlation) are acting to promote atomization. Specifically, as Eötvös number increases, the atomization onset Weber number decreases slightly.

As noted in Figure 5(b, c), buoyancy driven plumes (Ri > 0.4) accelerate after discharge, resulting in a narrowing of the plume and an increased velocity. We define local Re, Ri and We numbers, based on diameter and flow velocity at the onset of instability; see Figure 5(c). Figure 8 maps breakup regime on a scatter plot of local Richardson number and pipe discharge Richardson number. Here, the \( \text{Ri}_{\text{Local}} = \text{Ri}_{\text{Pipe}} \) line represents discharges that do not accelerate after leaving the discharge pipe. As the plot indicates, in no case did breakup occur in our data set until the flow had accelerated to the point where the local Ri was less than 0.4. Figure 9 shows breakup regimes plotted on local We and Eo numbers. It is
seen that no jet breakup occurs at a local We number below 10. Furthermore, the same
general trend in the atomization onset regime can be seen in Figure 9 and in Figure 7, with
critical Weber number decreasing slightly with increasing Eötvös number, although the local
parameters are at slightly higher values (90 < We < 300). The pinch-off regime occurs at
low local Eötvös numbers.

Following [11], Figure 10 exhibits Oh_{Pipe} versus Re_{Pipe} for our data set, where the
data points are illustrated by their corresponding breakup regimes. The correlation, Oh =
64/Re^{1.26}, suggested in [11] as the boundary for the atomization regime is also shown. The
foregoing correlation was suggested [11] based on a data set where Re \lesssim 1000 and Oh \gtrsim 0.05;
however, it is seen that the correlation applies relatively well to our data set too, which has
a broader range of dimensionless numbers than that in [11].

5.2 Computational simulation results

We employed the flow solver described in Section 3 and simulated the evolution of four oil
jets listed in Table 2, following the experimental results presented in Section 5.1. The Ri, We,
and Re numbers shown in Table 2 are based on the pipe diameter and the average velocity at
the pipe exit. The simulated jets vary from weakly-buoyant (J1) to buoyant (J4). The jets
are all in the laminar regime, except J4, which is slightly above the range of Re number for
laminar pipe flows (Re < 2300). In all cases, a laminar velocity profile was prescribed at the
pipe to match the experimental mass flow rate listed in Table 2. The size of the numerical
domain normalized by D_{pipe} was 10 \times 10 \times 20 for J1, J2, and J4 and 10 \times 10 \times 14 for J3. An
open boundary condition was imposed on the top surface of the numerical domain, where
the discharged jet leaves the domain. The side walls and the bottom surface of the domain
had no-slip boundary conditions. The number of grid points across the jet outlet diameter
was 20 in all simulations presented here. Due to prohibitive costs of grid refinement studies
in these jet simulations, we performed a grid coarsening study to ensure that 20 grid points
per diameter (GPD) provides adequate spatial resolution. We picked the J1 case, which exhibits the smallest features (i.e. surface instabilities, ligaments, and droplets) among all four jet cases. Performing simulations at resolutions of 10 and 14 GPD, we observed that 10 GPD does not provide enough resolution to capture the small features. While at 14 GPD the small features are captured, and they agree reasonably well with the 20 GPD results, they lack some of the fine spatial features observed at 20 GPD. Furthermore, as shown next, the results at 20 GPD resolution show very good agreement with the experimental results, which is another indication that 20 GPD is an adequate resolution.

Figures 11 to 14 present the simulation results for the J1 to J4 cases, respectively, along with the experimental images corresponding to each case (image a). For each jet, the 3D iso-surface of $f = 0.5$ (image d) and its mid-plane 2D cut view (image b) are illustrated; image (c) depicts the dimensionless vorticity field on the mid-plane 2D cut, where the oil-water interfaces are shown in red. The jet diameter and average outlet velocity were used to make the vorticity field dimensionless.

The simulations exhibit good agreement with the experimental results in all cases; the jet behavior, instabilities, and breakup regimes are captured well. The vorticity field images show axisymmetric ring vortices formed around the jet at locations where the Kelvin-Helmholtz instabilities deformed the jet. Such deformations are more frequent and occur closer to the jet outlet when the oil jet is most energetic, as seen in Figure 11 (J1 case: highest We; lowest R tête). In this case, the deformations result in small oil structures and drops formed near the jet body, which are seen in the experimental image 11(a) too. In other cases, the deformations evolve to skirt-like features on the jets. The interaction between the ambient fluid and the jet are clearly seen in the vorticity images; for example, image 13 shows vortices being shed off the skirt-like deformations. In the J3 and J4 cases, where $We < 100$, the jet breaks up at locations where the skirt-like deformations are formed, resulting in relatively large oil structures. Although deforming, the oil jets J3 and J4 exhibit a smooth
surface, compared to J1 and J2, which is due to relatively stronger surface tension effects. In the J4 case, large blobs of oil are formed and accelerated upward due to strong buoyancy effects, leaving wakes in the water, which are evident in image 14(c). Furthermore, in the J4 case, the local Ri numbers at the onset of breakup were computed using the simulation velocity field and local jet diameter at various times. The local Ri numbers were all below 0.4, which further supports the experimental observation discussed in Section 5.1.

Of importance is the lateral expansion of oil jets. Figure 15 shows the time-averaged dimensionless lateral expansion, $D_{\text{max}}/D_{\text{pipe}}$, along the dimensionless jet height, $z/D_{\text{pipe}}$ for J1 to J4. Note that from J1 to J4, the Richardson number increases while the Weber number decreases. As seen in Figure 15, the oil jet exhibits larger lateral expansion as Ri (and We) becomes smaller (and larger). That is because as the buoyancy and surface tension effects gain strength, the jet evolution becomes focused more along the axial direction due to a stronger buoyancy acceleration in the vertical direction.

6 Conclusions

The breakup of underwater buoyant oil jets and plumes was studied for a wide range of dimensionless numbers and viscosity ratios. The results show three main breakup regimes: atomization, skirt-type and pinch-off. Our experimental and computational results both suggest that the critical Weber number beyond which the jet starts to atomize is around 100 (the number is based on the jet discharge diameter). The critical We varies slightly with the jet Eötvös number; it is about 200 at low Eo numbers ($Eo < 10$) and about 80 at high Eo numbers. Furthermore, the Oh-Re relationship proposed in [11] as the onset of atomization regime, applies to our broader data set too; the jets for which $Oh > 64/Re^{1.26}$ at the outlet would atomize. Furthermore, it is observed in the experimental and computational results that the jet has to be accelerated to a point where the local Ri number becomes less than 0.4 for the breakup to occur. At that point, the local We number is above 10. This indicates
that the buoyancy effects enhance instabilities by accelerating the jet to a point where the
surface tension effects are overcome and the instabilities lead to jet breakup.

The computational simulations show formation of axisymmetric ring vortices around the
jet perimeter and their interactions with deformations caused by Kelvin-Helmholtz instabil-
ities. In energetic jets, these deformations can lead to ligaments that may break into small
droplets around the jet perimeter; in other cases, the deformations evolve to skirt-like fea-
tures that grow and lead to the breakup of the jet. Skirt-like features can only occur when
the surface tension is sufficiently strong. The computational results were used to obtain
time-averaged lateral expansion of jets under a range of Ri, We and Re numbers. Jets with
higher Ri (lower We) develop less lateral expansion because the jet is directed along the
direction of buoyancy force. The computational simulations presented here were performed
by using a GPU-accelerated MPI parallel two-phase flow solver, which allows for completing
a simulation 3 to 6 times faster compared to running the same simulation on CPUs only.
The acceleration factor becomes larger as the total number of grid points in a simulation
increases.

Finally, the flow rate at the Deepwater Horizon oil spill was estimated by McNutt et al. [12] to be 50,000 to 70,000 barrels per day, which results in the following estimates on the
dimensionless numbers of the jet: \(50,000 \lesssim Re \lesssim 70,000\); \(Oh \approx 1.30 \times 10^{-3}\); \(0.75 \lesssim Ri \lesssim 1.5\);
\(4,000 \lesssim We \lesssim 8,000\); \(Eo \approx 6,000\). Based on these estimates and Figure 10, the jet is in
the atomization regime as expected. Achieving the above ranges in our experimental or
computational simulations is currently prohibitively expensive.

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Table 1: Summary of experimental runs

<table>
<thead>
<tr>
<th>Nominal pipe diameters</th>
<th>1/4”, 3/4”, 1”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured pipe diameters</td>
<td>0.90 cm, 1.83 cm, 2.38 cm</td>
</tr>
<tr>
<td>Fluid 1</td>
<td>Jet viscosity: 0.65 cSt</td>
</tr>
<tr>
<td></td>
<td>Viscosity ratio: 0.22</td>
</tr>
<tr>
<td>Fluid 2</td>
<td>Jet viscosity: 5.0 cSt</td>
</tr>
<tr>
<td></td>
<td>Viscosity ratio: 1.67</td>
</tr>
<tr>
<td>Fluid 3</td>
<td>Jet viscosity: 20 cSt</td>
</tr>
<tr>
<td></td>
<td>Viscosity ratio: 6.7</td>
</tr>
<tr>
<td>All fluid pairs</td>
<td>Interfacial tension: 0.043 N/m</td>
</tr>
<tr>
<td>Parametric range</td>
<td>Viscosity ratio: 0.22, 1.67, 6.7</td>
</tr>
<tr>
<td></td>
<td>Reynolds number: 45.0×10^0 &lt; Re &lt; 4.5×10^4</td>
</tr>
<tr>
<td></td>
<td>Weber number: 3.2×10^{-1} &lt; We &lt; 4.2×10^4</td>
</tr>
<tr>
<td></td>
<td>Richardson number: 2×10^{-3} &lt; Ri &lt; 8.8×10^1</td>
</tr>
<tr>
<td></td>
<td>Eötvös number: 2×10^0 &lt; Eo &lt; 4.0×10^1</td>
</tr>
</tbody>
</table>
Table 2: Dimensional parameters and dimensionless numbers of simulated oil jets

<table>
<thead>
<tr>
<th>Jet No.</th>
<th>Nominal diameter of outlet (inch)</th>
<th>Oil viscosity (cSt)</th>
<th>Flow rate (L/s)</th>
<th>Ri</th>
<th>We</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>1/4</td>
<td>20</td>
<td>1.21×10^{-1}</td>
<td>3.42×10^{-3}</td>
<td>6.98×10^2</td>
<td>8.55×10^2</td>
</tr>
<tr>
<td>J2</td>
<td>3/4</td>
<td>20</td>
<td>2.03×10^{-1}</td>
<td>4.24×10^{-2}</td>
<td>2.34×10^2</td>
<td>7.07×10^2</td>
</tr>
<tr>
<td>J3</td>
<td>3/4</td>
<td>20</td>
<td>1.07×10^{-1}</td>
<td>1.53×10^{-1}</td>
<td>6.46×10^1</td>
<td>3.72×10^2</td>
</tr>
<tr>
<td>J4</td>
<td>1/4</td>
<td>0.65</td>
<td>1.36×10^{-2}</td>
<td>8.14×10^{-1}</td>
<td>7.10×10^0</td>
<td>2.97×10^3</td>
</tr>
</tbody>
</table>