Investment Analysis (FIN 670)
Fall 2009

Homework 5

Instructions: please read carefully

• You should show your work how to get the answer for each calculation question to get full credit
• The due date is Tuesday, November 10, 2009. Late homework will not be graded.

Name(s): Student ID
1. Compute the expected return for a three-stock portfolio with the following:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10%</td>
<td>20%</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>12%</td>
<td>28%</td>
<td>0.3</td>
</tr>
<tr>
<td>C</td>
<td>18%</td>
<td>40%</td>
<td>0.5</td>
</tr>
</tbody>
</table>

a. 13.3%
b. 14.6%
c. 29.3%
d. 32.4%

1. b \(10 \times 0.2 + 12 \times 0.3 + 18 \times 0.5 = 14.6\)

2. A portfolio is considered to be efficient if ______________.
   a. there is no other portfolio with a higher expected return
   b. there is no other portfolio with a lower risk
   c. there is no other portfolio offers a higher expected return with a higher risk
   d. there is no other portfolio offers a lower risk with the same expected return

2. d

3. Which of the following is (are) most correct concerning a two-stock portfolio?
   a. The portfolio should have no company specific risk.
   b. Portfolio standard deviation can never be a weighted average of the two stocks' standard deviations.
   c. Portfolio return is a weighted average of the two stocks' returns.
   d. All of the above are correct.

3. c

4. The maximum benefit of diversification can be achieved by combining securities in a portfolio where the correlation coefficient between the securities is ______________.
   a. between 0 and -1
   b. 0
   c. -1
   d. +1

4. c

5. A portfolio is composed of two stocks, A and B. Stock A has a standard deviation of return of 20% while stock B has a standard deviation of return of 30%. Stock A comprises 40% of the portfolio while stock B comprises 60% of the portfolio. What is the standard deviation of return on the portfolio if the correlation coefficient between the returns on A and B is 0.5?
   a. 23.1%
   b. 25%
   c. 26%
   d. 24.7%

5. a

\[
\sigma^2 = (.4)^2(.2)^2 + (.6)^2(.3)^2 + 2(.4)(.6)(.2)(.3)(.5)
\]

\[
\sigma^2 = .0532, \quad \sigma = .231
\]
6. A portfolio is composed of two stocks, A and B. Stock A has an expected return of 10% while stock B has an expected return of 18%. What is the proportion of stock A in the portfolio so that the expected return of the portfolio is 16.4%?
   a. 0.2  
   b. 0.8  
   c. 0.4  
   d. 0.6  
6. a \[ E(R_p) = (W_a)E(R_a) + (1-W_a)E(R_b) \]
   \[ 0.164 = W_a(0.10) + (1-W_a)(0.18) \]
   \[ W_a = 0.2 \]

7. Which of the following portfolios cannot lie on the efficient frontier?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>10%</td>
<td>15%</td>
</tr>
<tr>
<td>Y</td>
<td>12%</td>
<td>25%</td>
</tr>
<tr>
<td>Z</td>
<td>15%</td>
<td>25%</td>
</tr>
</tbody>
</table>

   a. Portfolio X  
   b. Portfolio Y  
   c. Portfolio Z  
   d. All portfolios should lie on the efficient frontier.

7. b

8. The standard deviation of return on stock A is 0.25 while the standard deviation of return on stock B is 0.30. If the covariance of returns on A and B is 0.06, the correlation coefficient between the returns on A and B is ______________.
   a. 0.2  
   b. 0.6  
   c. 0.7  
   d. 0.8  
8. d
   Correlation = \( \frac{0.06}{[0.25(0.30)]} = 0.8 \)

9. Careful selection of different stocks from different industries can eliminate the ______________ risk of a portfolio.
   a. Nonsystematic  
   b. Market  
   c. Total  
   d. All of the above.  
9. a

10. A positive covariance between two stocks' returns indicates that the two stocks' returns ______________.
    a. move in opposite direction  
    b. move in the same direction  
    c. have the same risk  
    d. have no relationship  
10. a
10. b

11. What happens typically to the portfolio's risk when more stocks are added to a 5-stock portfolio?
   a. The portfolio's market risk would decrease.
   b. The portfolio's total risk would decline.
   c. The portfolio's unsystematic would decrease.
   d. Both B and C above are correct.

11. d

12. Which of the following statements are correct concerning diversifiable risks?
   I. Diversifiable risks can be essentially eliminated by investing in several unrelated securities.
   II. The market rewards investors for diversifiable risk by paying a risk premium.
   III. Diversifiable risks are generally associated with an individual firm or industry.
   IV. Beta measures diversifiable risk.
   a. I and III only
   b. II and IV only
   c. I and IV only
   d. II and III only
   e. I, II, and III only

12. a

13. Which of the following statements concerning nondiversifiable risk are correct?
   I. Nondiversifiable risk is measured by standard deviation.
   II. Systematic risk is another name for nondiversifiable risk.
   III. The risk premium increases as the nondiversifiable risk increases.
   IV. Nondiversifiable risks are those risks you can not avoid if you are invested in the financial markets.
   a. I and III only
   b. II and IV only
   c. I, II, and III only
   d. II, III, and IV only
   e. I, II, III, and IV

13. d

14. Which one of the following is an example of a nondiversifiable risk?
   a. a well respected president of a firm suddenly resigns
   b. a well respected chairman of the Federal Reserve suddenly resigns
   c. a key employee of a firm suddenly resigns and accepts employment with a key competitor
   d. a well managed firm reduces its work force and automates several jobs
   e. a poorly managed firm suddenly goes out of business due to lack of sales

14. b
15. Which of the following risks are relevant to a well-diversified investor?
   I. systematic risk  
   II. unsystematic risk  
   III. market risk  
   IV. nondiversifiable risk  
   a. I and III only  
   b. II and IV only  
   c. II, III, and IV only  
   d. I, II, and IV only  
   e. I, III, and IV only  

15. e

16. Which one of the following is an example of systematic risk?
   a. the price of lumber declines sharply  
   b. airline pilots go on strike  
   c. the Federal Reserve increases interest rates  
   d. a hurricane hits a tourist destination  
   e. people become diet conscious and avoid fast food restaurants  

16. c

17. Which one of the following is an example of unsystematic risk?
   a. the inflation rate increases unexpectedly  
   b. the federal government lowers income taxes  
   c. an oil tanker runs aground and spills its cargo  
   d. interest rates decline by one-half of one percent  
   e. the GDP rises by 2 percent more than anticipated  

17. c

18. Which of the following actions help eliminate unsystematic risk in a portfolio?
   I. spreading the retail industry portion of a portfolio over five separate stocks  
   II. combining stocks with bonds in a portfolio  
   III. adding some international securities into a portfolio of U.S. stocks  
   IV. adding some U.S. Treasury bills to a risky portfolio  
   a. I and III only  
   b. I, II, and IV only  
   c. I, III, and IV only  
   d. II, III, and IV only  
   e. I, II, III, and IV  

18. e
19. All things equal, diversification is most effective when
   a. securities' returns are positively correlated.
   b. securities' returns are uncorrelated.
   c. securities' returns are high.
   d. securities' returns are negatively correlated.
   e. A and C.

d
All things equal, diversification is most effective when securities' returns are negatively correlated.

20. An investor who wishes to form a portfolio that lies to the right of the optimal risky portfolio on the Capital Allocation Line must:
   a. lend some of her money at the risk-free rate and invest the remainder in the optimal risky portfolio.
   b. borrow some money at the risk-free rate and invest in the optimal risky portfolio.
   c. such a portfolio cannot be formed.
   d. invest only in risky securities.
   e. B and D

e
An investor who wishes to form a portfolio that lies to the right of the optimal risky portfolio on the Capital Allocation Line must borrow some money at the risk-free rate and invest in the optimal risky portfolio. This also implies that the investors must invest only in risky securities.

21. Portfolio theory as described by Markowitz is most concerned with
   a. the elimination of systematic risk.
   b. the identification of unsystematic risk.
   c. the effect of diversification on portfolio risk.
   d. active portfolio management to enhance returns.
   e. none of the above

c
Portfolio theory as described by Markowitz is most concerned with the effect of diversification on portfolio risk.

22. Security C has expected return of 12% and standard deviation of 20%. Security D has expected return of 15% and standard deviation of 27%. If the two securities have a correlation coefficient of 0.7, what is their covariance?
   a. 0.038
   b. 0.070
   c. 0.018
   d. 0.013
   e. 0.054

a
\[ \text{Cov}(r_X, r_Y) = (.7)(.20)(.27) = .0378. \]
The following information is for question 23-26

Abigail Grace has a $900,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $100,000. Her financial advisor provided her with the following estimates:

The correlation coefficient of ABC stock returns with the original portfolio returns is 0.40.

23. The inheritance changes Grace’s overall portfolio and she is deciding whether to keep the ABC stock. Assuming Grace keeps the ABC stock, calculate the:

i. Expected return of her new portfolio which includes the ABC stock.

\[ E(r_{NP}) = w_{OP} E(r_{OP}) + w_{ABC} E(r_{ABC}) = (0.9 \times 0.67) + (0.1 \times 1.25) = 0.728\% \]

ii. Covariance of ABC stock returns with the original portfolio returns.

\[ \text{Cov} = r \times \sigma_{OP} \times \sigma_{ABC} = 0.40 \times 2.37 \times 2.95 = 2.7966 \approx 2.80 \]

iii. Standard deviation of her new portfolio which includes the ABC stock.

\[ \sigma_{NP} = \left[ w_{OP}^2 \sigma_{OP}^2 + w_{ABC}^2 \sigma_{ABC}^2 + 2 w_{OP} w_{ABC} (\text{Cov}_{OP,ABC}) \right]^{1/2} \]
\[ = \left[ (0.9^2 \times 2.37^2) + (0.1^2 \times 2.95^2) + (2 \times 0.9 \times 0.1 \times 2.80) \right]^{1/2} \]
\[ = 2.2673\% \approx 2.27\% \]
24. If Grace sells the ABC stock, she will invest the proceeds in risk-free government securities yielding 0.42 percent monthly. Assuming Grace sells the ABC stock and replaces it with the government securities, calculate the:

(remember the standard deviation of government securities = 0)

a. Expected return of her new portfolio which includes the government securities.

\[ E(r_{NP}) = w_{OP} E(r_{OP}) + w_{GS} E(r_{GS}) = (0.9 \times 0.67) + (0.1 \times 0.42) = 0.645\% \]

b. Covariance of the government security returns with the original portfolio returns.

\[ Cov = r \times \sigma_{OP} \times \sigma_{GS} = 0 \times 2.37 \times 0 = 0 \]

c. Standard deviation of her new portfolio which includes the government securities.

\[ \sigma_{NP} = \left[ w_{OP}^2 \sigma_{OP}^2 + w_{GS}^2 \sigma_{GS}^2 + 2 w_{OP} w_{GS} (Cov_{OP, GS}) \right]^{1/2} \]
\[ = \left[ (0.9^2 \times 2.37^2) + (0.1^2 \times 0) + (2 \times 0.9 \times 0.1 \times 0) \right]^{1/2} \]
\[ = 2.133\% \approx 2.13\% \]

25. Determine whether the systematic risk of her new portfolio, which includes the government securities, will be higher or lower than that of her original portfolio. Explain briefly

Adding the risk-free government securities would result in a lower beta for the new portfolio. The new portfolio beta will be a weighted average of the individual security betas in the portfolio; the presence of the risk-free securities would lower that weighted average.
26. Based on conversations with her husband, Grace is considering selling the $100,000 of ABC stock and acquiring $100,000 of XYZ Company common stock instead. XYZ stock has the same expected return and standard deviation as ABC stock. Her husband comments, “It doesn’t matter whether you keep all of the ABC stock or replace it with $100,000 of XYZ stock.” State whether her husband’s comment is correct or incorrect. Justify your response.

The comment is not correct. Although the respective standard deviations and expected returns for the two securities under consideration are equal, the covariances between each security and the original portfolio are unknown, making it impossible to draw the conclusion stated. For instance, if the covariances are different, selecting one security over the other may result in a lower standard deviation for the portfolio as a whole. In such a case, that security would be the preferred investment, assuming all other factors are equal.

27. George Stephenson’s current portfolio of $2.0 million is invested as follows:

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Value</th>
<th>Percent of Total</th>
<th>Expected Annual Return</th>
<th>Annual Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds</td>
<td>$200,000</td>
<td>10%</td>
<td>4.6%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Domestic large-cap equities</td>
<td>600,000</td>
<td>30%</td>
<td>12.4%</td>
<td>19.5%</td>
</tr>
<tr>
<td>Domestic small-cap equities</td>
<td>1,200,000</td>
<td>60%</td>
<td>16.0%</td>
<td>29.9%</td>
</tr>
<tr>
<td>Total Portfolio</td>
<td>$2,000,000</td>
<td>100%</td>
<td>13.8%</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

Stephenson soon expects to receive an additional $2.0 million and plans to invest the entire amount in an index fund that best complements the current portfolio. Stephanie Coppa, CFA, is evaluating the four index funds shown in the following table for their ability to produce a portfolio that will meet two criteria relative to the current portfolio: (1) maintain or enhance expected return and (2) maintain or reduce volatility.

Each fund is invested in an asset class that is not substantially represented in the current portfolio.

<table>
<thead>
<tr>
<th>Fund</th>
<th>Expected Annual Return</th>
<th>Expected Annual Standard Deviation</th>
<th>Correlation of Returns with Current Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15%</td>
<td>25%</td>
<td>+0.80</td>
</tr>
<tr>
<td>B</td>
<td>11%</td>
<td>25%</td>
<td>+0.60</td>
</tr>
<tr>
<td>C</td>
<td>16%</td>
<td>25%</td>
<td>+0.90</td>
</tr>
<tr>
<td>D</td>
<td>14%</td>
<td>22%</td>
<td>+0.65</td>
</tr>
</tbody>
</table>

State which fund Coppa should recommend to Stephenson. Justify your choice by describing how your chosen fund best meets both of Stephenson’s criteria. No calculations are required.

Fund D represents the single best addition to complement Stephenson’s current portfolio, given his selection criteria. First, Fund D’s expected return (14.0 percent) has the
potential to increase the portfolio’s return somewhat. Second, Fund D’s relatively low correlation with his current portfolio (+0.65) indicates that Fund D will provide greater diversification benefits than any of the other alternatives except Fund B. The result of adding Fund D should be a portfolio with approximately the same expected return and somewhat lower volatility compared to the original portfolio.

The other three funds have shortcomings in terms of either expected return enhancement or volatility reduction through diversification benefits. Fund A offers the potential for increasing the portfolio’s return, but is too highly correlated to provide substantial volatility reduction benefits through diversification. Fund B provides substantial volatility reduction through diversification benefits, but is expected to generate a return well below the current portfolio’s return. Fund C has the greatest potential to increase the portfolio’s return, but is too highly correlated to provide substantial volatility reduction benefits through diversification.

The following data apply to question 28--32

A pension fund manager is considering 3 mutual funds. This first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund that yields a rate of 8%. The probability distribution of the risky funds is as follows

<table>
<thead>
<tr>
<th></th>
<th>Expected return</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock fund (S)</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>Bond fund (B)</td>
<td>12%</td>
<td>15%</td>
</tr>
</tbody>
</table>

The correlation between the fund returns is 0.10.

28. What are the investment proportions in the minimum variance portfolio of the two risky funds, what is the expected value and standard deviation of its rate of return? (The weight of stock in the minimum variance portfolio is calculated using the following formula)

\[
 w_{\text{Min}}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)}
\]

The parameters of the opportunity set are:

\[ E(r_S) = 20\%, \ E(r_B) = 12\%, \ \sigma_S = 30\%, \ \sigma_B = 15\%, \ \rho = 0.10 \]

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that Cov(r_S, r_B) = \rho \sigma_S \sigma_B]:

<table>
<thead>
<tr>
<th></th>
<th>Bonds</th>
<th>Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>225</td>
<td>45</td>
</tr>
<tr>
<td>Stocks</td>
<td>45</td>
<td>900</td>
</tr>
</tbody>
</table>

The minimum-variance portfolio is computed as follows:
w_{\text{Min}}(S) = \frac{\sigma_s^2 - \text{Cov}(r_s, r_B)}{\sigma_s^2 + \sigma_B^2 - 2\text{Cov}(r_s, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739

w_{\text{Min}}(B) = 1 - 0.1739 = 0.8261

The minimum variance portfolio mean and standard deviation are:

E(r_{\text{Min}}) = (0.1739 \times 20) + (0.8261 \times 12) = 13.39% 

\sigma_{\text{Min}} = [(w_s^2 \sigma_s^2 + w_B^2 \sigma_B^2 + 2w_s w_B \text{Cov}(r_s, r_B))]^{1/2} 

= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2} 

= 13.92%

29. Now, assume you decide to include T-bill into your complete portfolio. Solve numerically for the proportions of each asset and for the expected return and standard deviation of the optimal risky portfolio (the optimal risky portfolio is the combination of stock and bond fund that gives investor the best risk-return trade-off when combining with T-bill)

(The proportion of the optimal risky portfolio invested in the stock fund is given by:

\[ w_s = \frac{[E(r_s) - r_f] \sigma_s^2 - [E(r_B) - r_f] \text{Cov}(r_s, r_B)}{[E(r_s) - r_f] \sigma_s^2 + [E(r_B) - r_f] \sigma_B^2 - (E(r_B) - r_f + E(r_B) - r_f) \text{Cov}(r_s, r_B)} \]

The proportion of the optimal risky portfolio invested in the stock fund is given by:

\[ w_s = \frac{[E(r_s) - r_f] \sigma_s^2 - [E(r_B) - r_f] \text{Cov}(r_s, r_B)}{[E(r_s) - r_f] \sigma_s^2 + [E(r_B) - r_f] \sigma_B^2 - (E(r_B) - r_f + E(r_B) - r_f) \text{Cov}(r_s, r_B)} \]

\[ = \frac{[(20 - 8) \times 225] - [(12 - 8) \times 45]}{[(20 - 8) \times 225] + [(12 - 8) \times 900] - [(20 - 8 + 12 - 8) \times 45]} = 0.4516 \]

w_B = 1 - 0.4516 = 0.5484

The mean and standard deviation of the optimal risky portfolio are:

E(r_P) = (0.4516 \times 20) + (0.5484 \times 12) = 15.61%

\sigma_p = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2} 

= 16.54%
30. What is the reward-to-variability ratio of the best feasible CAL?

\[
\frac{E(r_p) - r_f}{\sigma_p} = \frac{15.61 - 8}{16.54} = 0.4601
\]

31. Suppose now that your portfolio must yield an expected return of 14% and be efficient, that is, on the best feasible CAL.

a. What is the standard deviation of your portfolio?

If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

\[
E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_p} \sigma_c = 8 + 0.4601 \sigma_c
\]

Setting \(E(r_C)\) equal to 14%, we find that the standard deviation of the optimal portfolio is 13.04%.

b. What is the proportion invested in the T-bill fund and each of the two risky funds?

To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let \(y\) be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

\[
E(r_C) = (1 - y)r_f + yE(r_p) = r_f + y[E(r_p) - r_f] = 8 + y(15.61 - 8)
\]

Setting \(E(r_C) = 14\%\) we find: \(y = 0.7884\) and \((1 - y) = 0.2116\) (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

- Proportion of stocks in complete portfolio = \(0.7884 \times 0.4516 = 0.3560\)
- Proportion of bonds in complete portfolio = \(0.7884 \times 0.5484 = 0.4324\)
32. If you were to use only the two risky funds and still require an expected return of 14%, what would be the investment proportions of your portfolio? Compare its standard deviation to that of the optimal portfolio in the previous problem (#31). What do you conclude?

Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (wS) and the appropriate proportion in the bond fund (wB = 1 - wS) as follows:

\[ 14 = 20w_S + 12(1 - w_S) = 12 + 8w_S \Rightarrow w_S = 0.25 \]

So the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

\[ \sigma_P = \left[ (0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45) \right]^{1/2} = 14.13\% \]

This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.

33. Stocks offer an expected rate of return of 10% with a standard deviation of 20%, and gold offers an expected return of 5% with a standard deviation of 25%.

a. In light of the apparent inferiority of gold to stocks with respect to both mean return and volatility, would anyone hold gold? Explain your answer

Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a part of a portfolio. If the correlation between gold and other stocks is sufficiently low (or negative), gold will be held as a component in a diversified portfolio,

b. How would you answer (a) if the correlation coefficient between gold and stocks were 1.0? Could these expected returns, standard deviations, and correlation represent an equilibrium for the security market (i.e., if no one chooses to hold gold, would the situation be there for long time)?

If the correlation between gold and stocks equals +1, then no one would hold gold since gold would move exactly in the same direction of stocks and stocks are better than gold in both return and standard deviation.

Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.