Homework 6

Instructions: please read carefully

- You should show your work how to get the answer for each calculation question to get full credit
- The due date is Tue, November 17, 2009. Late homework will not be graded.

Name(s):       Student ID
1. According to the CAPM:
   a. The expected return of a stock will be doubled if its beta increases from 1 to 2.
   b. High-beta stocks should have higher risk premium to systematic risk ratios.
   c. A stock's risk premium depends on its beta.
   d. All of the above are correct.
1. c

2. If a stock's expected return plots under the Security Market Line, then the stock should have _____________.
   a. a negative alpha
   b. a positive alpha
   c. a small beta
   d. both market risk and company specific risk
2. a

3. According to the SML, a stock's return is expected to ____________ if the stock has a beta of 1.5 and the market return is expected to increase by 2%.
   a. decline by 2%
   b. rise by 2%
   c. decline by 3%
   d. rise by 3%
3. d

4. The Security Market Line represents the relationship between _____________.
   a. the total risk and expected return on a security
   b. the nonsystematic risk and expected return on a security
   c. the systematic risk and expected return on a security
   d. None of the above.
4. c

5. The expected return on a stock with a beta of 1.5 is 15%. If the expected risk-free rate of return is 3%, what should be the market risk premium?
   a. 5%
   b. 8%
   c. 12%
   d. 15%
5. b

6. If the CAPM is valid, is the following situation possible?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Standard Deviation</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30%</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>25%</td>
<td>1.2</td>
</tr>
</tbody>
</table>
   a. Possible.
   b. Not possible.
6. a

7. The expected return on the market is 12%. The expected return on a stock with a beta of 1.5 is 17%. What is the risk-free rate of return according to the CAPM?
   a. 2%
b. 4%
c. 5%
d. 8%

7. a
17 = Rf + 1.5(12 – Rf)
Rf = 2

8. You invest $8,000 in stock A with a beta of 1.4 and $12,000 in stock B with a beta of 0.8. The beta of this formed portfolio is _____________.
   a. 1.04
   b. 1.14
   c. 2.04
   d. 2.20
8. a
Beta(p) = 0.4*1.4 + 0.6*0.8 = 1.04

9. A Stock has an estimated (forecasted) rate of return of 15.5% and a beta of 1.5. The market expected rate of return is 10% and the risk-free rate is 3%. The alpha of the stock is _____________.
   a. -2%
   b. 0%
   c. 2%
   d. 3%
9. c
Alpha = 15.5 – [3+1.5*(10-3)]=2

10. According to the CAPM, overvalued securities should have _____________.
    a. large betas
    b. positive alphas
    c. zero alphas
    d. negative alphas
10. d

11. In Fama and French's three-factor model, ____________ and ____________ are added to the market index model to explain stock returns.
    a. firm size; firm revenues
    b. firm size; book value to market value ratio
    c. firm sales; market value to book value ration
    d. firm sales; firm cost of capital
11. b

12. The real world data always support the CAPM.
   a. True
   b. False
12. b
The following information is for questions 13-14

Karen Kay, a portfolio manager at Collins Asset Management, is using the CAPM for making recommendations to her clients. Her research department has developed the information shown as follows:

<table>
<thead>
<tr>
<th></th>
<th>Forecasted return (%)</th>
<th>Standard deviation (%)</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock X</td>
<td>14</td>
<td>36</td>
<td>0.8</td>
</tr>
<tr>
<td>Stock Y</td>
<td>17</td>
<td>25</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The expected return of market is $E(R_m) = 14\%$, risk-free rate $r_f = 5\%$

13. Using CAPM, calculate expected return and alpha for each stock

$E(R_x) = 5 + 0.8(14-5) = 12.2$
$E(R_y) = 5 + 1.5(14-5) = 18.5$
Alpha(x) = 14 – 12.2 = 1.8 > 0
Alpha(y) = 17-18.5 = -1.5 < 0

14. Which stock is overvalued, which is undervalued?

X is undervalued since its forecasted return is higher that its required return
Y is overvalued since its forecasted return is lower than its required return

The following information is for questions 15-16

Karen Kay, a portfolio manager at Collins Asset Management, is using the CAPM for making recommendations to her clients. Her research department has developed the information shown as follows:

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation (%)</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock X</td>
<td>36</td>
<td>0.8</td>
</tr>
<tr>
<td>Stock Y</td>
<td>25</td>
<td>1.5</td>
</tr>
</tbody>
</table>
15. Identify and justify which stock would be more appropriate for an investor who wants to add this stock to a well-diversified portfolio (assume the investor wants to reduce the risk of his portfolio)

Choose stock X since in well-diversified portfolio, X is less riskier than Y (lower beta). In a diversified portfolio, only market risk (beta) matter since the specific is diversified away.

16. Identify and justify which stock would be more appropriate for an investor who wants to hold this stock as a single-stock portfolio (assume the investor wants to reduce the risk of his portfolio)

Choose Y since in a single-stock portfolio, which is not diversified, investors will face the total risk that includes both market and specific risk. Y has lower standard deviation (total risk).

17. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected return (%)</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>1.4</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>1.2</td>
</tr>
</tbody>
</table>

No, not possible, since in CAPM world, the investors are assumed to diversify their portfolio, and then the only risk they have is market risk (measured by beta). They get higher return because of higher beta. In this case, higher beta get lower return, so it is not correct.

18. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected return (%)</th>
<th>standard deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
<td>25</td>
</tr>
</tbody>
</table>

Yes, it is possible since in CAPM, the investors are assumed to diversify their portfolio, and then the only risk they have is market risk (measured by beta). They get higher return because of higher beta, not because of higher standard deviation. It is possible that while stock A has higher standard deviation (which is higher total risk), but when investors put stock A in a diversified portfolio, stock A might have lower beta because more specific risk component is diversified away.
19. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Market</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>A</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market, which is not possible according to the CAPM, since the CAPM predicts that the market portfolio is the most efficient portfolio. Using the numbers supplied:

\[ S_A = \frac{16 - 10}{12} = 0.5 \]

\[ S_M = \frac{18 - 10}{24} = 0.33 \]

These figures imply that Portfolio A provides a better risk-reward tradeoff than the market portfolio.

20. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Market</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

No, it is not possible, since market is an efficient portfolio, however, in here, clearly portfolio A dominates the market portfolio.
21. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>10%</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>18</td>
<td>1.0</td>
</tr>
<tr>
<td>A</td>
<td>16</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Not possible. Given these data, the SML is: $E(r) = 10\% + \beta(18\% - 10\%)$

A portfolio with beta of 1.5 should have an expected return of:

$E(r) = 10\% + 1.5 \times (18\% - 10\%) = 22\%$

The expected return for Portfolio A is 16\% so that Portfolio A plots below the SML (i.e., has an alpha of −6\%), and hence is an overpriced portfolio. This is inconsistent with the CAPM.

22. If CAPM is valid, is the following situation possible? Explain briefly

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>10%</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>18</td>
<td>1.0</td>
</tr>
<tr>
<td>A</td>
<td>16</td>
<td>.9</td>
</tr>
</tbody>
</table>

Not possible. The SML is the same as in Problem 21. Here, the required expected return for Portfolio A is: $10\% + (0.9 \times 8\%) = 17.2\%$

This is still higher than 16\%. Portfolio A is overpriced, with alpha equal to: −1.2\%

23. Which one of the following portfolios should have the most systematic risk?
   a. 50 percent invested in U.S. Treasury bills and 50 percent in a market index mutual fund
   b. 20 percent invested in U.S. Treasury bills and 80 percent invested in a stock with a beta of .80
   c. 10 percent invested in a stock with a beta of 1.0 and 90 percent invested in a stock with a beta of 1.40
   d. 100 percent invested in a mutual fund which mimics the overall market
   e. 100 percent invested in U.S. Treasury bills

(Hint: compute the beta of the portfolios and then compare the betas. Also, as known, beta of T-bill = 0, beta of market or market index or market index mutual fund = 1)
a. \( \text{Beta}(p) = 0.5 \)

b. \( \text{Beta}(p) = 0.64 \)

c. \( \text{Beta}(p) = 1.36 \) (highest)

d. \( \text{Beta}(p) = 1 \)

e. \( \text{Beta}(p) = 0 \)

24. Your portfolio consists of $100,000 invested in a stock which has a beta = 0.8, $150,000 invested in a stock which has a beta = 1.2, and $50,000 invested in a stock which has a beta = 1.8. The risk-free rate is 7 percent. Last year this portfolio had a required rate of return of 13 percent. This year nothing has changed except for the fact that the market risk premium has increased by 2 percent (two percentage points). What is the portfolio's current required rate of return?

- a. 5.14%
- b. 7.14%
- c. 11.45%
- d. 15.33%
- e. 16.25%

\[
\text{\( b_p = \frac{100,000}{300,000} \cdot 0.8 + \frac{150,000}{300,000} \cdot 1.2 + \frac{50,000}{300,000} \cdot 1.8 \)}}
\]

\[
\text{\( b_p = 1.167 \).
}\]

Last year: \( r = 13 \%
\)
\[
13\% = 7\% + \text{RP}_m(1.167)
\]
\[
6\% = \text{RP}_m(1.167)
\]
\[
\text{RP}_m = 5.1429\%.
\]

This year:
\[
r = 7\% + (5.1429\% + 2\%) \cdot 1.167
\]
\[
r = 15.33\%.
\]

25. A money manager is managing the account of a large investor. The investor holds the following stocks:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Amount Invested</th>
<th>Estimated Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2,000,000</td>
<td>0.80</td>
</tr>
<tr>
<td>B</td>
<td>$5,000,000</td>
<td>1.10</td>
</tr>
<tr>
<td>C</td>
<td>$3,000,000</td>
<td>1.40</td>
</tr>
<tr>
<td>D</td>
<td>$5,000,000</td>
<td>??</td>
</tr>
</tbody>
</table>
The portfolio’s required rate of return is 17 percent. The risk-free rate, rRF, is 7 percent and the return on the market, rM, is 14 percent. What is Stock D’s estimated beta?

a. 1.256  
b. 1.389  
c. 1.429  
d. 2.026  
e. 2.154

25. d

Portfolio beta is found from the CAPM:
17% = 7% + (14% - 7%)bp
bp = 1.4286.

The portfolio beta is a weighted average of the betas of the stocks within the portfolio.

1.4286 = (2/15)(0.8) + (5/15)(1.1) + (3/15)(1.4) + (5/15)bd
1.4286 = 0.1067 + 0.3667 + 0.2800 + (5/15)bd
0.6752 = 5/15bd
bd = 2.026.

26. You want to create a portfolio equally as risky as the market, and you have $1,000,000 to invest. Given this information, fill in the rest of the following table

<table>
<thead>
<tr>
<th>Stock</th>
<th>Investment</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>$180,000</td>
<td>0.75</td>
</tr>
<tr>
<td>Stock B</td>
<td>$290,000</td>
<td>1.30</td>
</tr>
<tr>
<td>Stock C</td>
<td>?</td>
<td>1.45</td>
</tr>
<tr>
<td>Risk-free asset</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

wa = $180,000 / $1,000,000 = .18
wb = $290,000 / $1,000,000 = .29

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:
\[ \beta_p = 1.0 = w_A(0.75) + w_B(1.30) + w_C(1.45) + w_{Rf}(0) \]

Solving for the weight of Stock C, we find:

\[ w_C = 0.33655172 \]

So, the dollar investment in Stock C must be:

Invest in Stock C = 0.33655172($1,000,000) = $336,551.72

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

\[ 1 = w_A + w_B + w_C + w_{Rf} \]
\[ 1 = 0.18 + 0.29 + 0.33655172 + w_{Rf} \]
\[ w_{Rf} = 0.19344828 \]

So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = 0.19344828($1,000,000) = $193,448.28

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

\[ 1 = w_A + w_B + w_C + w_{Rf} \]
\[ 1 = 0.20 + 0.25 + 0.3433 + w_{Rf} \]
\[ w_{Rf} = 0.206667 \]

So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = 0.206667($1,000,000) = $206,667
27. You have been provided the following data about the securities of three firms, the market portfolio, and the risk-free asset

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected return</th>
<th>Standard deviation</th>
<th>Correlation*</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>0.10</td>
<td>0.27</td>
<td>(i)</td>
<td>0.85</td>
</tr>
<tr>
<td>Stock B</td>
<td>0.14</td>
<td>(ii)</td>
<td>0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Stock C</td>
<td>0.17</td>
<td>0.70</td>
<td>0.35</td>
<td>(iii)</td>
</tr>
<tr>
<td>Market portfolio</td>
<td>0.12</td>
<td>0.20</td>
<td>(iv)</td>
<td>(v)</td>
</tr>
<tr>
<td>Risk-free asset</td>
<td>0.05</td>
<td>(vi)</td>
<td>(vii)</td>
<td>(viii)</td>
</tr>
</tbody>
</table>

*Correlation indicates the correlation between a security and the market portfolio. For example, the correlation between stock B and market portfolio is 0.50

a. Fill in the missing values in the table (i.e., find out (i), (ii), ..., (viii)

Hint: the formula to calculate beta is

\[
\beta = \frac{\text{Cov}(r_i, r_m)}{\sigma_i^2} = \frac{\rho(r_i, r_m)\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho(r_i, r_m)\sigma_i}{\sigma_m}
\]

\(\rho(r_i, r_m)\) is the correlation between stock i and market portfolio

27. a. (i) We can use the equation to calculate beta, we find:

\[
\beta_A = \frac{(\rho_{A,M})(\sigma_A)}{\sigma_M} = 0.85 = \frac{(\rho_{A,M})(0.27)}{0.20}
\]

\(\rho_{A,M} = 0.63\)

(ii) Using the equation to calculate beta, we find:

\[
\beta_B = \frac{(\rho_{B,M})(\sigma_B)}{\sigma_M} = 1.50 = \frac{(0.50)(\sigma_B)}{0.20}
\]

\(\sigma_B = 0.60\)

(iii) Using the equation to calculate beta, we find:

\[
\beta_C = \frac{(\rho_{C,M})(\sigma_C)}{\sigma_M}
\]
\[ \beta_C = (.35)(.70) / 0.20 \]

\[ \beta_C = 1.23 \]

(iv) The market has a correlation of 1 with itself.

(v) The beta of the market is 1.

(vi) The risk-free asset has zero standard deviation.

(vii) The risk-free asset has zero correlation with the market portfolio.

(viii) The beta of the risk-free asset is 0.

b. Using the CAPM to find the expected return of the stock, we find:

*Firm A:*

\[
E(R_A) = R_f + \beta_A[E(R_M) - R_f]
\]

\[
E(R_A) = 0.05 + 0.85(0.12 - 0.05)
\]

\[ E(R_A) = .1095 \text{ or } 10.95\% \]

According to the CAPM, the expected return on Firm A’s stock should be 10.95 percent. However, the expected return on Firm A’s stock given in the table is only 10 percent. Therefore, Firm A’s stock is overpriced, and you should sell it.

*Firm B:*

\[
E(R_B) = R_f + \beta_B[E(R_M) - R_f]
\]

\[
E(R_B) = 0.05 + 1.5(0.12 - 0.05)
\]

\[ E(R_B) = .1550 \text{ or } 15.50\% \]

According to the CAPM, the expected return on Firm B’s stock should be 15.50 percent. However, the expected return on Firm B’s stock given in the table is 14 percent. Therefore, Firm B’s stock is overpriced, and you should sell it.

*Firm C:*

\[
E(R_C) = R_f + \beta_C[E(R_M) - R_f]
\]

\[
E(R_C) = 0.05 + 1.23(0.12 - 0.05)
\]

\[ E(R_C) = .1358 \text{ or } 13.58\% \]

According to the CAPM, the expected return on Firm C’s stock should be 13.58 percent. However, the expected return on Firm C’s stock given in the table is 17 percent. Therefore, Firm C’s stock is underpriced, and you should buy it.
28. Consider the following information about Stock I and II

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability</th>
<th>Stock I</th>
<th>Stock II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.15</td>
<td>0.09</td>
<td>-0.30</td>
</tr>
<tr>
<td>Normal</td>
<td>0.55</td>
<td>0.42</td>
<td>0.12</td>
</tr>
<tr>
<td>Irrational exuberance</td>
<td>0.30</td>
<td>0.26</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Assume the market is in equilibrium, i.e., the forecast $E(R) = \text{required } E(R)$. The market risk premium is 7.5%, the risk-free rate is 4%. Which stock has most systematic risk? Which one has most unsystematic risk? Which stock is riskier? Explain.

The amount of systematic risk is measured by the $\beta$ of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the $\beta$ of the asset. The expected return of Stock I is:

$$E(R_I) = 0.15(0.09) + 0.55(0.42) + 0.30(0.26) = 0.3225 \text{ or } 32.25\%$$

Using the CAPM to find the $\beta$ of Stock I, we find:

$$0.3225 = 0.04 + 0.075\beta_I$$

$$\beta_I = 3.77$$

The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock’s variance, we find:

$$\sigma_I^2 = 0.15(0.09 - 0.3225)^2 + 0.55(0.42 - 0.3225)^2 + 0.30(0.26 - 0.3225)^2$$

$$\sigma_I^2 = 0.01451$$

$$\sigma_I = (0.01451)^{1/2} = 0.1205 \text{ or } 12.05\%$$

Using the same procedure for Stock II, we find the expected return to be:

$$E(R_{II}) = 0.15(-0.30) + 0.55(0.12) + 0.30(0.44) = 0.1530$$

Using the CAPM to find the $\beta$ of Stock II, we find:

$$0.1530 = 0.04 + 0.075\beta_{II}$$

$$\beta_{II} = 1.51$$

And the standard deviation of Stock II is:

$$\sigma_{II}^2 = 0.15(-0.30 - 0.1530)^2 + 0.55(0.12 - 0.1530)^2 + 0.30(0.44 - 0.1530)^2$$
\[ \sigma_{II}^2 = .05609 \]

\[ \sigma_{II} = (.05609)^{1/2} = .2368\text{ or }23.68\% \]

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I’s. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.